Epistemic Trust and Education: Effects of Informant Reliability on Student Learning of Decimal Concepts

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The epistemic trust literature emphasizes that children’s evaluations of informants’ trustworthiness affects learning, but there is no evidence that epistemic trust affects learning in academic domains. The current study investigated how reliability affects decimal learning. Fourth and fifth graders (N = 122; M_age = 10.1 years) compared examples from consistently accurate and inaccurate informants (consistent) or informants who were each sometimes accurate and inaccurate (inconsistent). Fourth graders had higher conceptual knowledge and fewer misconceptions in the consistent condition than the inconsistent condition, and vice versa for fifth graders due to differences in prior exposure to decimals. Given the same examples, learning differed depending on informant reliability. Thus, epistemic trust is a malleable factor that affects learning in an academic domain.

Human culture is striking in the degree to which we systematically create and engage in situations in which we obtain information from other people (Csibra & Gergely, 2006). Many have argued that although learning from other people introduces great challenges, it may also explain our unique ability to accumulate knowledge over generations (Csibra & Gergely, 2009; Tomasello, Carpenter, Call, Behne, & Moll, 2005). For these reasons, there is a growing consensus in the psychology literature that reasoning about people as sources of evidence is a critical aspect of cognition and learning (Sobel & Kushnir, 2013; for review, see Mills, 2013).

Research in psychology has studied how evaluations of informants’ trustworthiness affect learning (e.g., Koenig & Harris, 2005; Pasquini, Corriveau, Koenig, & Harris, 2007). An informant is a person who provides information, and informant reliability can be judged by the accuracy of an informant’s information. Epistemic trust research often focuses on informants’ accuracy in word learning, where children are presented with informants who give correct and incorrect labels for familiar objects, followed by a final trial to learn the label for a novel object. This paradigm allows manipulation of reliability via correct and incorrect examples, and assessment of children’s tracking of, and inferences based on, informants’ information, and this paradigm has often been used with preschoolers. The evidence suggests that children as young as 3 years old can track accuracy and selectively learn from previously accurate informants over previously inaccurate informants (e.g., Ganea, Koenig, & Millett, 2011; Koenig & Harris, 2005). Additionally, elementary school-aged children will determine whether informants are trustworthy using cues such as an informant’s accuracy, expertise, intent, informativeness, capability, and biases, although this work has not used the traditional paradigm described earlier (e.g., Danovitch & Keil, 2007; Gweon, Pelton, Konopka, & Schulz, 2014; Mills & Grant, 2009; Mills & Landrum, 2012). This work has shown that information provided by informants affects learners’ beliefs about whether informants are trustworthy (Birch, Vauthier, & Bloom, 2008; Corriveau & Harris, 2009; Gweon, Pelton, & Schulz, 2011; Koenig & Harris, 2005; Pasquini et al., 2007), and that learners’ beliefs about informants affect inferences about new information (Corriveau, Fusaro, & Harris, 2009; Landrum, Mills, & Johnston, 2013; Lutz & Keil, 2002; for review, see Mills, 2013). In addition, past research has emphasized that children as young as 3 and 4 years old evaluate explanations from others to determine whether causal, noncircular language was used, and they use these evaluations to determine the credibility of sources (e.g., Bernard, Mercier, & Clément, 2012; Corriveau...
Although education research has not systematically manipulated informants’ trustworthiness, it has investigated the effects of contrasting correct and incorrect examples for learning (e.g., Große & Renkl, 2007). Research emphasizes the benefits of incorrect examples, especially as a means of illustrating misconceptions (Durkin & Rittle-Johnson, 2012; Siegler, 2002; VanLehn, 1999), and the benefits of comparison, especially as a means of highlighting shared relational structures and abstract commonalities (e.g., Kotovsky & Gentner, 1996; Loewenstein, Thompson, & Gentner, 1999). This work has emphasized that the efficacy of incorrect examples and comparison may depend on their timing during instruction and students’ prior experiences (e.g., Große & Renkl, 2007; Rittle-Johnson, Star, & Durkin, 2009). However, education research has not focused on how the evaluation of the sources of examples might influence learning.

Critically, academic learning differs from areas typically studied in the epistemic trust literature. In academic learning, learners are not expected to have mastered the domain before seeing examples, whereas in typical epistemic trust paradigms, learners first learn about informants through examples for which they already know the correct answer. To investigate the potential role of epistemic trust in academic learning, we investigate learning at two stages: before and after students have experience in the domain.

There are two interesting possibilities for how experience may affect learning from informants of varying reliability. To explain, it is first worth revisiting the standard methods used in the epistemic trust and comparison of worked examples literature. In the epistemic trust literature, learners are often presented with two verbal labels, one correct and the other incorrect, for a familiar object. This literature makes explicit the presence of two informants who provide the information, and this allows manipulation of an informant’s accuracy—and therefore trustworthiness—over trials. Each informant can be consistently correct, consistently incorrect, or inconsistent. In the worked examples literature, learners are often presented with two worked examples, each labeled as correct or incorrect, but studies vary in whether they make the presence of two informants explicit. We marry these paradigms by having two informants provide labeled worked examples over a number of trials. It is then possible to manipulate whether the pair of informants is consistent—where one informant is always accurate and the other always inaccurate—or inconsistent—where both are inconsistent and equally (in)accurate.

How might consistency and inconsistency affect learning? First, having consistently trustworthy (or not) informants may reduce complexity of the situation, and thus consistency may always facilitate learning. Alternatively, there may be cases when inconsistent informants are most beneficial, and theoretical and computational accounts converge on this prediction. Inconsistency introduces an added source of information that may facilitate or inhibit learning. Cognitive load theory argues that learning is best when both overload and underload are avoided (Paas, Renkl, & Sweller, 2004). Similarly, computational modeling of epistemic trust has shown that children’s behavior is consistent with an expectation that informants tend to be consistently accurate or not (Eaves & Shafto, 2012; Shafto, Eaves, Navarro, & Perfors, 2012), and computational approaches have long argued that a balance between predictability and unpredictability is important for learning (Kidd, Piantadosi, & Aslin, 2012; Shannon, 1948; Tummeltshammer & Kirkham, 2013). Thus, theoretical and computational accounts converge on the prediction that less experienced students may show greater learning from consistent informants, whereas more experienced students may show greater learning from inconsistent informants. This difference depending on students’ experience may be especially important to consider when learning from incorrect examples. Incorrect examples may need to be presented from the same informant for less experienced students who need more predictability, but incorrect examples may need to be presented from different informants for more experienced students who need more unpredictability.

Current Study

We examined students’ learning in the domain of decimal fractions (i.e., decimals). An understanding of decimals is necessary for algebra proficiency and advanced mathematics (e.g., National Mathematics Advisory Panel, 2008), and is a significant predictor of later mathematics achievement (Siegler et al., 2012). Unfortunately, children and adults often have difficulty with decimals because of persistent misconceptions involving decimal magnitude (e.g., Desmet, Gregoire, & Mussolin, 2010; Glasgow, Ragan, Fields, Reys, & Wasman, 2000; Resnick et al., 1989; Rittle-Johnson, Siegler, & Alibali, 2001).
These misconceptions make decimals an ideal domain for studying the effects of informant reliability.

Several measures were used to assess children’s inferences and learning. Following past epistemic trust research, children’s inferences about informants were assessed using ask and explicit judgment questions (e.g., Koenig & Harris, 2005). Ask questions were used to determine whom children would ask for information in the future (e.g., “Who would you ask for help on this kind of decimal question in the future?” and “Who would you ask for help on an algebra problem in the future?”). Explicit judgment questions were used to determine whom children would judge as more knowledgeable (e.g., “Who knows more about decimals?” and “Who knows more about math?”). On the basis of worked examples research, we assessed decimal learning using knowledge gains from pretest to posttest, posttest scores, reduction in errors due to misconceptions, and proportion of errors due to misconceptions at posttest (Durkin & Rittle-Johnson, 2012; Große & Renkl, 2007; Rittle-Johnson et al., 2001). Across all analyses, we assessed effects of grade (fourth grade, no formal instruction vs. fifth grade, some formal instruction) and effects of condition (consistent vs. inconsistent informants).

**Method**

**Participants**

Consent was obtained from 124 fourth and fifth graders from an urban parochial school in Kentucky. Data were collected in November 2013. These students were primarily Caucasian (96%) and all spoke English as their primary language at home. They were also generally from middle- to upper-class families with only 3% of students at the school eligible for free and reduced lunch. Two students were excluded because they already mastered the skills taught during the training—they solved 90% or more of the procedural knowledge items correctly at pretest. The final sample consisted of 122 students (68 female) with 76 in fourth grade and 46 in fifth grade. The average age was 10.1 years (range = 9.1–11.7 years). Each grade had one mathematics teacher and three classrooms, and the classes rotated through the teacher’s mathematics classroom each day. The fourth graders had only studied decimals in the context of money in the classroom, and the fifth graders had a few formal lessons on decimals at the beginning of the school year. Thus, by working with participants in fourth and fifth grades, we could investigate how past formal instruction might affect learning from consistent and inconsistent informants.

**Design**

Students participated in a pretest–training–posttest design. For the training session, students were randomly assigned to the consistent condition (n = 61) or the inconsistent condition (n = 61). During this session, students received a 3-min introductory lesson on decimals and then completed a packet with 12 pairs of worked examples and corresponding explanation prompts. Each pair illustrated two different procedures (one correct and one incorrect) for placing a decimal on a number line from 0 to 1, with each procedure labeled as correct or incorrect accordingly. Thus, students were told whether each example they saw was correct or incorrect (see Figure 1 for a sample page). Students in the consistent condition always saw a correct example provided by the same informant and an incorrect example provided by another informant. In the inconsistent condition, students saw the exact same examples and explanation prompts as the consistent condition, but each informant was correct only half of the time throughout the training.

**Materials**

**Training Packet**

Students were told a cover story about identical twins from another school (the informants) who were asked to place decimals on number lines. Each worked example showed where each informant placed a decimal on a number line and an explanation of his reasoning (Figure 1). Each page of the packet contained a question prompting students to reflect on and compare the two examples, which students answered aloud. To answer these explanation prompts, students needed to consider both the correct and incorrect examples. In the consistent condition, one informant always generated the correct examples, and another informant always generated the incorrect examples. In the inconsistent condition, each informant generated correct examples about half of the time and incorrect examples the other half of the time. Throughout the packet, all students solved 13 practice problems to improve attention to the worked examples (Atkinson, Derry, Renkl, & Wortham, 2000).

Three correct solution procedures were illustrated across the worked examples (Rittle-Johnson
et al., 2001). Three incorrect solution procedures were illustrated, based on common decimal misconceptions (Desmet et al., 2010; Resnick et al., 1989): (a) treating decimals like whole numbers (0.25 is greater than 0.7 because 25 is greater than 7), (b) misunderstanding the role of zero by ignoring zeros in the tenths place and adding magnitude for zeros on the end (0.08 is equal to 0.8 and 0.320 is greater than 0.32), and (c) thinking of decimals like the denominator of common fractions so that shorter decimals are larger (0.341 must be less than 0.3 because 0.341 contains smaller parts).

Finally, at the end of the training packet, students were asked to answer questions about the informants. They answered four ask questions assessing which informant they would like to ask for help in the future. They also answered six explicit judgment questions assessing whether they thought each informant was knowledgeable and whom they thought was more knowledgeable (e.g., Koenig & Harris, 2005).

Assessment

The assessment used for the pretest and posttest measured conceptual and procedural knowledge and misconceptions and was adapted from one used in past work (Durkin & Rittle-Johnson, 2012, 2015; Rittle-Johnson et al., 2001). Conceptual knowledge is defined as the ability to recognize and understand key domain concepts. Procedural knowledge is defined as the ability to execute action sequences to solve problems. These two distinct knowledge types have been used frequently in past research (e.g., Canobi, Reeve, & Pattison, 2003). Sample items of each type are shown in Table 1, including the number of items of each type. Internal consistency was calculated for the items on the pretest and posttest, and was good for conceptual knowledge (αs = .91 and .89, respectively), procedural knowledge (αs = .79 and .85), whole number misconceptions (αs = .79 and .84), role of zero misconceptions (αs = .72 and .71), and fraction misconceptions (αs = .72 and .88).

Procedure

Students completed the pretest as a group in a 20-min session in their classrooms. The students then participated in a one-on-one session where they completed the appropriate training packet. Students spent about the same amount of time on the training across conditions and grades. On average, fourth graders spent 22.68 min (SD = 4.38) on the training in the consistent condition and 22.26 min (SD = 4.58) in the inconsistent condition, and fifth graders spent 23.00 min (SD = 4.11) in the consistent condition and 21.26 min (SD = 3.79) in the inconsistent condition. Immediately after the training, students completed the posttest.

Coding

Assessment

Items were scored for accuracy using the criteria specified in Table 1. Students’ answers were coded for the three misconception errors. The proportions of misconception errors, across conceptual and procedural knowledge items, were calculated by dividing the number of misconception errors made on all items by the total possible number of misconception errors of that type.
Ask and Explicit Judgment Questions

For all participants, we scored responses to each ask question as a 1 if they chose to ask the more accurate informant for help in the future (in the consistent condition or that same informant in the inconsistent condition) and a 0 if they chose to ask the less accurate informant for help. Explicit judgment questions were scored similarly. We then added the scores of each of these questions for an overall ask question score and explicit judgment question score.

Results

Pretest Knowledge

There were no significant differences between conditions in procedural knowledge, so we do not report those results. Students’ conceptual knowledge of decimal magnitude, referred to as “knowledge” from now on, was variable at pretest with prevalent misconception errors. To check for differences in knowledge between conditions at pretest, we ran analysis of variance (ANOVA) models with condition and grade as between-subjects factors. There were no significant differences between conditions, $F(1, 119) = 0.47$, $p = .497$, $\eta^2_p = .004$. There was a significant difference between grades, $F(1, 119) = 78.88$, $p < .001$, $\eta^2_p = .399$. The fifth graders performed better on these measures than the fourth graders (see Figure 2).

Did Children Track Informant Reliability?

Data Analysis

To check whether students tracked informants’ accuracy, we investigated students’ ask and explicit judgment scores. We compared results in each condition to chance using one-sample $t$ tests. We then conducted independent-samples $t$ tests to check that students in the consistent condition were more likely than students in the inconsistent condition to (a) ask the correct informant (from the consistent condition) for help in the future and (b) judge the correct informant as more knowledgeable than the incorrect informant.
Ask Questions

Students answered *ask* questions that assessed who they would rather receive help from in the future. Students in the *consistent* condition showed a significant preference for the correct informant (*M* = 3.15 of 4 times, *SD* = 0.81), *t*(60) = 11.02, *p* < .001, *d* = 2.85), whereas students in the *inconsistent* condition did not show a significant preference (*M* = 1.92 of 4 times, *SD* = 0.69), *t*(60) = −0.93, *p* = .357, *d* = −0.24). The difference between conditions was also significant, *t*(120) = 9.00, *p* < .001, *d* = 1.64. Effect of condition was independent of age and grade.

Explicit Judgment Questions

Students were asked *explicit judgment* questions about which informant they thought was more knowledgeable. Students in the *consistent* condition showed a significant preference for the correct informant (*M* = 5.21 of 6, *SD* = 0.14), *t*(60) = 15.51, *p* < .001, *d* = 4.01), whereas students in the *inconsistent* condition did not show a significant preference (*M* = 3.15 of 6 times, *SD* = 0.20), *t*(60) = 0.73, *p* = .471, *d* = 0.19). The difference between conditions was significant, *t*(120) = 8.35, *p* < .001, *d* = 1.52. Effect of condition was independent of age and grade.

These results indicate that students tracked whether the one informant was correct more often than the other informant, and students used this information when choosing whom to trust for information.

**Did Informant Reliability Affect Student Learning and Knowledge?**

Due to the amount of variation in pretest scores, particularly for fifth graders, our analyses focused on assessing the effects of informant reliability relatively. First, we examined students’ *learning* via prettest to posttest *gain scores* using a Condition × Grade ANOVA. Second, we examined effects on students’
posttest knowledge via estimated marginal means that account for differences in pretest knowledge with a Condition × Grade analysis of covariance using pretest scores as covariates. We also initially included students’ scores on the ask and explicit judgment questions as covariates in the model, but because they were never significant predictors of our outcomes, we removed them to create more parsimonious models. Table 2 reports correlations among the dependent measures, showing that they provide different assessments of learning and knowledge. These provide converging measures regarding the effect of informant reliability.

First, for knowledge gain scores, there were no significant effects of condition, $F(1, 118) = 0.04, p = .835, \eta_p^2 < .001$, or grade, $F(1, 118) = 0.01, p = .939, \eta_p^2 < .001$. However, there was a significant interaction between condition and grade, $F(1, 118) = 5.98, p = .016, \eta_p^2 = .048$. This interaction indicated that students in fourth grade had greater gains in knowledge in the consistent condition, but students in fifth grade had greater gains in the inconsistent condition (Figure 3a).

Second, we looked at differences in students’ posttest scores accounting for pretest scores. Again, there were no significant effects of condition, $F(1, 116) = 0.03, p = .870, \eta_p^2 < .001$, or grade, $F(1, 116) = 0.45, p = .506, \eta_p^2 = .004$. There was a significant interaction between condition and grade, $F(1, 116) = 4.53, p = .035, \eta_p^2 = .038$. Students in fourth grade had higher posttest scores in the consistent condition, but students in fifth grade had higher posttest scores in the inconsistent condition (Figure 3b).

Third, we examined the reduction in each misconception type from pretest to posttest. For whole number misconception errors, there was no significant effect of condition, $F(1, 118) = 0.10, p = .752, \eta_p^2 = .001$, but there was a significant effect of grade such that fourth graders had a greater reduction in this misconception than fifth graders, $F(1, 118) = 4.67, p = .033, \eta_p^2 = .038$. There was no significant interaction between condition and grade (Figure 3c), $F(1, 118) = 1.65, p = .202, \eta_p^2 = .014$. For role of zero misconception errors, there was no significant effect of condition, $F(1, 118) = 0.39, p = .534, \eta_p^2 = .003$, or grade, $F(1, 118) = 0.27, p = .603, \eta_p^2 = .002$. There was a significant interaction between condition and grade, $F(1, 118) = 7.14, p = .009, \eta_p^2 = .057$. For fourth graders, role of zero misconception errors decreased most in the consistent condition, and fifth graders had greater reduction in the role of zero misconception in the inconsistent condition (Figure 3d). For fraction misconception errors, there was no significant effect of condition, $F(1, 118) = 0.04, p = .836, \eta_p^2 < .001$, but there was a significant effect of grade such that fourth graders had a greater increase in this misconception than fifth graders, $F(1, 118) = 10.54, p = .002, \eta_p^2 = .082$. There was no significant interaction between condition and grade, $F(1, 118) = 0.32, p = .576, \eta_p^2 = .003$. The fraction misconception errors likely increased because this is a misconception type that students in the United States sometimes transition to after realizing that the whole number and role of zero misconceptions are incorrect (Durkin & Rittle-Johnson, 2015).

Fourth, we analyzed the proportion of misconception errors students made at posttest accounting for pretest. For whole number misconception errors, there was no significant effect of condition, $F(1, 116) = 0.30, p = .586, \eta_p^2 = .003$, or grade, $F(1, 116) = 3.37, p = .069, \eta_p^2 = .028$. Consistent with previous analyses, there was a significant interaction between condition and grade, $F(1, 116) = 5.59, p = .020, \eta_p^2 = .046$, such that fourth graders were less likely to have whole number misconceptions in
the consistent condition, and fifth graders were less likely to have whole number misconceptions in the inconsistent condition (Figure 3e). For role of zero misconception errors, there was no significant effect of condition, $F(1, 116) = 1.20, p = .275, \eta^2_p = .010$, or grade, $F(1, 116) = 0.01, p = .929, \eta^2_p < .001$, but there was a significant interaction between condition and grade, $F(1, 116) = 8.20, p = .005, \eta^2_p = .066$. Again, fourth graders were less likely to have role of zero misconceptions in the consistent condition, and fifth graders were less likely to have role of zero misconceptions in the inconsistent condition (Figure 3f). For fraction misconception errors, there was no significant effect of condition, $F(1, 116) = 0.04, p = .845, \eta^2_p < .001$, or grade, $F(1, 116) = 0.84, p = .361, \eta^2_p = .007$, and no significant interaction between condition and grade, $F(1, 116) = 0.32, p = .571, \eta^2_p = .003$. 

Figure 3. Effects of condition and grade on (a) knowledge gains, (b) knowledge posttest scores, (c) reduction in whole number misconceptions, (d) reduction in role of zero misconceptions, (e) whole number misconceptions at posttest, and (f) role of zero misconceptions at posttest. Values for figures a, c, and d are means with standard error bars, and figures b, e, and f are estimated marginal means with standard error bars.
Summary

Across all analyses, several findings emerged. Fourth graders had larger knowledge gains, higher knowledge scores, greater reduction in role of zero misconception errors, and fewer whole number and role of zero misconception errors in the consistent condition than in the inconsistent condition. The reverse was true for fifth graders. Thus, the consistent condition was best for students in fourth grade, who had no formal decimal instruction in class, while the inconsistent condition was best for students in fifth grade, who had some formal decimal instruction. Consequently, manipulating epistemic trust via informant reliability may facilitate or impede learning, depending on prior instruction.

Discussion

Psychology research has emphasized the importance of epistemic trust and informant reliability for children’s learning but has not focused on academic domains. Meanwhile, research in education has investigated the value of correct and incorrect examples but has not systematically manipulated informants’ history of accuracy. The results from the current study indicate that given the exact same examples, children’s learning varies depending on whether the pair of informants is consistent or inconsistent. Depending on learners’ experience, they may need different levels of predictability or unpredictability to improve their understanding. Less experienced learners seem to learn best when it is clear which informant is trustworthy, and this predictability and lower cognitive load may help learners accurately process information and form new conceptual structures. More experienced learners seem to learn best when they need to process which informant is accurate on a given trial, and this unpredictability and higher cognitive load may force learners to engage with examples more than when they can defer to others’ trustworthiness. This may help these learners strengthen conceptual knowledge and modify their previous misconceptions. Thus, the results show that epistemic trust affects student learning in educationally relevant ways.

When does trust elicit positive effects on learning? We tested children at two different stages: early in instruction and later, after they had experience in the domain. Our results imply that consistent informants do not lead to uniformly better learning. Past research has found mixed results on whether students with low prior knowledge can benefit from studying incorrect examples (e.g., Durkin & Rittle-Johnson, 2012; Große & Renkl, 2007). Durkin and Rittle-Johnson (2012) had different informants present examples from trial to trial, whereas Große and Renkl (2007) provided examples without any information about the informants. Thus, variation in informant context may have contributed to the mixed results. Consequently, instructors may want to vary consistency according to learners’ experience when teaching with incorrect examples.

This result is broadly consistent with a variety of theoretical and computational accounts of learning (Eaves & Shafto, 2012; Kidd et al., 2012; Paas et al., 2004; Shafto et al., 2012; Shannon, 1948; Tummelshammer & Kirkham, 2013). These accounts propose that titrating the novelty of situations—and thus cognitive load or predictability—has important implications for learning. More specifically, instructors may want to consider how informant trustworthiness can be leveraged to improve learning depending on students’ prior experience. We may also want to identify the precise mechanism at work when children are learning from consistent and inconsistent informants using additional measures, such as general mathematics achievement, metacognitive skills, and verbal intelligence. This is an important direction for future work. The epistemic trust literature has compiled an impressive list of conditions under which children do or do not trust informants. Accuracy is one key factor in determining trustworthiness, but there are others, including expertise, authority, helpfulness, deception, and negativity (for review, see Mills, 2013). Future research will be necessary to determine whether these factors can be manipulated to affect learning. In addition, future work should determine whether children are making broad, global judgments about informants’ trustworthiness or if they only used these worked examples to make judgments about informants’ trustworthiness in math domains. Finally, past work indicates that children attend to causal language when determining an informant’s trustworthiness (e.g., Bernard et al., 2012) and specific manipulation of this factor is an interesting direction for future work.

Our results are the first, to the best of our knowledge, which show effects of trustworthiness on learning in an academically relevant domain. The results are tantalizing in that they imply trustworthiness is not always good, but may instead represent a malleable factor by which we may affect educational goals. Future work will be required to
establish the precise mechanisms by which trustworthiness affects learning and effectiveness in classroom pedagogy, but these results provide reason to be optimistic about leveraging the growing literature on epistemic trust toward practical gains in student learning.

References


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